SEOUL ICM 2014

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John Milnor – Abel Lecture

Master's mathematical vision embraces humanity

John Milnor, a professor of mathematics at Stony Brook University, U.S.A., is one of the best known mathematicians to Koreans partly because of his sweep of major awards in mathematics. He won the Fields Medal in 1962, the Wolf Prize in 1989 and the Abel Prize in 2011.

The Norwegian Academy of Science and Letters bestowed the Abel Prize on him for his lifetime achievements in topology, geometry and algebra.

The 83-year-old scholar, who is also known for his gift of communicating difficult mathematics to general audiences, gave the Abel Lecture on Friday, Aug. 15, during the SEOUL ICM 2014.

The same day, he sat down for an interview moderated by Jongil Park, a professor of mathematics at Seoul National University.

John Milnor, a professor of mathematics at Stony Brook University in the United States, at an interview on Friday, shortly before his Abel lecture at the SEOUL ICM 2014.

Q. You won three major awards in mathematics. Which prize are you most proud of?

A. It's very nice to be appreciated; I don't want to compare. I got the prizes at different times. I suppose it was more startling when I was young, but more wonderful to be appreciated when I was elderly.

You proved what is now known as the Fary-Milnor Theorem in knot theory at a young age. When did you start working on complex mathematics?

This was a problem that the professor described in class, and it was very intriguing. I had been studying mathematics for a few years at that point and had some idea of work methods. There is always more to learn in mathematics. Like the Red Queen, we have to keep running.

You are currently focused on dynamical systems. Could you explain it in layman's term?

Dynamics is a very general subject, which can be applied in many different areas. I certainly don't want to make it sound as if I am contributing to all of these areas. I work in a narrow part of the field, just trying to understand very simple mathematical models that give rise to very complex behaviors.

Many physical systems are extremely complicated. The only way to analyze them is by experiments with very fast and large computers, but one can often get ideas of general principles from studying much simpler systems. That is the approach I take, driven by mathematical curiosity and trying to understand some of the simplest systems. Even though they look simple, they can induce very complicated behaviors which are difficult to understand.

You are known as one of the best writers among mathematicians. Many engineering and science students in Korea have difficulty in writing. How did you become a good writer?

Well, most of my writing has been driven by my desire to understand something. In order to understand something, I have to write it down. If I write it down clearly enough that I can understand it, then other people will be able to understand it. But, it takes a great deal of care to translate an idea into words. So, typically I have to write things over and over again.

Why is mathematics important?

I think it was the physicist Eugene Wigner who said that mathematics seems to miraculously describe things in the real world. Often, mathematics is developed before it is needed, and turns out to be useful afterwards. At other times, mathematics has been developed because it was clearly needed for a given purpose.

Achievements

John Milnor spent his undergraduate and graduate student years at Princeton University, studying knot theory under the supervision of Ralph Fox. He has worked in many areas of mathematics, including game theory, differential geometry, algebraic and differential topology, algebraic K-theory and dynamical systems.

His profound ideas in mathematics have had a great influence on the mathematical community from the second half of the 20th century until now. One of his most celebrated works is his proof in 1956 of the existence of 7-dimensional spheres with several nonstandard differentiable structures. In 1959, he showed that the diffeomorphism classes of oriented exotic spheres of any dimension form the nontrivial elements of an abelian monoid under connected sum, which is a finite abelian group if the dimension is not 4. (An exotic sphere is a differentiable manifold M that is homeomorphic but not diffeomorphic to the standard Euclidean n-sphere.) The classification of exotic spheres by Michel Kervaire and Milnor in 1963 showed that the oriented exotic 7-spheres are the non-trivial elements of a cyclic group of order 28 under the operation of connected sum.

Subsequently, he worked on the topology of isolated singular points of complex hypersurfaces in general, developing the theory of the Milnor fibration, whose fiber has the homotopy type of a bouquet of μ spheres, where μ is known as the Milnor number. All of his work in topology, geometry and algebra displays features of great research: salient facts, profound insights, vivid imagination, striking surprises and supreme beauty.

Numerous mathematical concepts, results and conjectures are named after him, including the Fary-Milnor theorem, Milnor exact sequence, Hilton-Milnor theorem, Milnor conjecture in algebraic K-theory and also in knot theory, Milnor construction, Milnor K-theory, Milnor fibration, Milnor number and Milnor sphere.

Seven volumes of his collected papers have been published by the American Mathematical Society.

He has also written many influential books, which include "Differential Topology," "Morse Theory," "Lectures on the h-Cobordism Theorem," "Singular Points of Complex Hypersurfaces," "Introduction to Algebraic K-Theory," "Dynamics in One Complex Variable" and "Characteristic Classes" (with J. Stasheff).

Compiled by
Dae-Woong Lee
Chonbuk National University

Invited ICM Panel 2 – 'How should we teach better?'

Teaching and learning manifest mutual benefit

Invited panelists discussed the topic "How should we teach better?" at the 2014 International Congress of Mathematicians yesterday afternoon. The panelists of Panel 2 were William Barton of the University of Auckland, New Zealand; Jean-Marie Laborde of the Université Joseph Fourier, France; and Man Keung Siu of the University of Hong Kong, Hong Kong, China. Deborah Ball, University of Michigan, U.S.A., and Sunwook Hwang of Soongsil University compiled the panel's comments before the meeting; Barton moderated the panel discussion; Ball was unable to attend.

At the Invited Panel 2 session yesterday, from left: Jean Marie Laborde, Université Joseph Fourier, France; William Barton, University of Auckland, New Zealand; and Man Keung Siu, University of Hong Kong, Hong Kong, China.

Q. Mathematics is a broad discipline, and many think the study load must be lightened. But mathematics is also widely applied in many fields, so others warn against weakening secondary-school education in the discipline. Is one opinion wrong, or can the differences be harmonized?

A. László Lovász, a past IMU president, said in a paper in 2007 that mathematics has broadened beyond any possibility of having a reasonably sized core mathematics curriculum, even at the secondary-school level. Many mathematicians would say that it does not matter so much what students learn so long as they have learned something in depth; that is, that they have experienced deep learning – its excitements and pleasures, its methods and conventions, its difficulty and the need for persistence. That is the resolution of the conundrum. We need to focus more on mathematical processes and less on mathematical content. There are not so many basic concepts that should be learned in primary and secondary schools, and these basic concepts come up time and again at all levels.

Most people would agree that training in both mathematics and pedagogy are equally important for mathematics teachers. Which of the two do you consider more important to teach mathematics better? What are the critical factors in raising teachers' professionalism?

Both mathematics and educational techniques are important, but the balance is not the same at different levels and in different contexts. There are two critical factors.

First, all mathematics teachers, at whatever level, must be practicing mathematicians. At an appropriate level they must love mathematics and actively engage in mathematical activities themselves. To acquire self-confidence in coming to grips with the subject matter, they must know their own shortcomings and how to eliminate them. They should be a role model in studying and reflecting on the discipline, so all mathematics teachers should engage in research. That research will be different in form and scope than the kind a working research mathematician engages in, but it should be similar in spirit. A school teacher has to explain mathematics in language and at a level of sophistication suitable to the knowledge, background and mental development of the pupils.

A second critical factor is that all mathematics teachers must constantly develop their abilities, both mathematically and pedagogically.

The eight core competencies for future society are self-understanding, self-management, information literacy, career development, communication, ethical practicality, critical reasoning and creative problem solving skills. In mathematics, what core competencies are there other than problem solving, communication and reasoning skills?

All eight of the stated core competencies must present in a mathematically educated person, and there is no reason why mathematics should be excluded in the course of developing these competencies. One benefit of studying mathematics that is seldom emphasized in the Western community is its value in moral and ethical education. This had already been pointed out by Xu Guang-qi, who first translated "Elements" into Chinese in collaboration with the Italian Jesuit Matteo Ricci in 1607. In the twentieth century, the late Russian mathematics educator Igor Sharygin once said, "Learning mathematics builds up our virtues, sharpens our sense of justice and our dignity and strengthens our innate honesty and our principles. The life of mathematical society is based on the idea of proof, one of the most highly moral ideas in the world."

Recently, there has been growing criticism that moving towards an activity-based classroom is a waste of time. At the elementary school level, students seem not to know why they are doing certain things and have no understanding of the meaning or introspective reasoning involved. But this surely cannot mean that only textbook content should be presented in classrooms and that learning activities should be abandoned altogether. What type of curriculum can use activities to deliver the material more effectively?

In view of the comments on the first question, an activity-based classroom must be a significant part of all mathematics education. The preoccupation with standards testing has led to content-focused education that is, as explained above, ultimately futile. This is not to suggest that no content be taught, but any system that has little or no place for the development of skills in mathematical processes will be doing its students a serious disservice.

In Korea, there are efforts to integrate many academic disciplines, such as the creation of a STEAM program (including A for the arts). What are the ultimate objectives in an educational curriculum that advocates STEM or STEAM? What do we need to consider in contrast to existing mathematical curricula to emphasize interdisciplinary reasoning and competence?

Education systems should, in general, lead to developing broadly educated people rather than specialists with no other interests. This does not preclude some students from concentrated studies in one area, but this is likely to occur later in their education when they have a specific goal in mind, and certainly not during compulsory education. There is no single definition of "broad education," but complementarities among disciplines, appropriateness of different types of thinking and expertise, historical and social links between ideas, and variety within any one discipline (including mathematics) are important principles when designing curricula in compulsory education. In the general framework of STEAM, the history of mathematics may play an important role, but this has to be understood in a broad sense, that is, the evolution of mathematical ideas and knowledge, the men and women who were responsible for them, the times and climate which nurtured (or perhaps stifled) them, and the impact and influence they exerted upon contemporary society.

Final words from each panelist:

Deborah Ball: Teaching is recipient-oriented. Teachers must be people who are fascinated with mathematics and even more fascinated with other people's thinking about mathematics. Teaching requires being deeply engaged in the cultivation of mathematical interest and capability in others rather than being impressed with one's own. Teachers must be people who believe fundamentally in mathematics as a democratic and broadly human activity into which all can enter, not one reserved for an elite few.

William Barton: As Plutarch said, "The mind is not a vessel to be filled, but a fire to be kindled."

Jean-Marie Laborde: Echoing Bert Waits, the initiator and founder of Teacher Teaching with Technology (T^3), I would like to stress that a deep understanding of mathematics comes with the problem solving in which students have to engage. Man Keung Siu: As an ancient Chinese saying puts it, "Teaching and learning help each other." A good teacher must like the subject and also like his or her students.

Compiled by

Deborah Ball
University of Michigan

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Soongsil University

Martin Hairer: Fields Medalist

Theory puts meaning to terms in SPDE

Martin Hairer was not among the frontrunners before the announcement of the 2014 Fields Medal, as the 38-year-old Austrian said himself at an interview on Friday.

But the professor of mathematics at the University of Warwick in England has made a significant contribution to the study of stochastic partial differential equations. He said his career has been motivated by curiosity and a yearning to unlock the mysteries of mathematics.

The interview was moderated by Panki Kim of SNU and Minhyong Kim of Oxford University.

Q. Please explain to the general public what your work is about.

A. (Laughing) That is always a difficult one. Stochastic partial differential equations describe the evolution of systems that depend on space and time and have some random components. In some interesting situations, there are equations that you can write according to classical rules, but in the mathematical sense they don't really have any meaning because we don't know how to interpret some of the terms that appear in the equation. So I built a general theory describing how to systematically provide meaning for these terms. It is a system for providing mathematically rigorous meaning to some equations that you want to use because they are natural equations, but which somehow don't make sense.

Can you give an example of where your theory can be applied?

One example would be that of a magnet. At room temperature, a magnet has a magnetic field. If you heat it up, at some point it loses its magnetic field. There is a critical temperature above which the magnet becomes neutral; it does not behave like a magnet anymore but like a normal piece of iron. If you are interested in what happens inside the magnet when you are very close to this critical temperature, the magnetic field has a very large random fluctuation instead of being nicely aligned in one direction.

An equation that describes this is one of the equations that I mentioned in my lecture yesterday.

Martin Hairer, a professor of mathematics at the University of Warwick and one of the four 2014 Fields Medal recipients, talks to reporters at COEX on Friday.

What is your next goal?

The theory I developed and that I received the medal for is extremely young. The main article in which the theory is described is not yet in print. So for next few years, I want to explore further in this direction. There are many open questions.

I think it is good for any mathematician to change research fields every now and then, but it is difficult to predict far in advance when that should happen. It depends on the ideas floating around.

What is the most important thing in doing mathematics? Good funding, good teachers, or good ideas?

Mostly good ideas. I think that for most people, it is important to have some level of funding, but many pure mathematicians are quite happy with relatively small amounts of funding, just enough for them to have the freedom to invite colleagues, maintain collaboration and go to conferences.

I think there is an unfortunate tendency nowadays for granting agencies to want to pick winners and then shower the winners with money and not give anything to others. That is not how the mathematical community really works.

Concerning the difference between mathematics and physics, I'll quote your famous paper "A Theory of Regularity Structures." It says,"This allows us, for the first time, to give a mathematically rigorous meaning to many interesting stochastic PDEs arising in physics." That sounds important and exciting to a mathematician, but some physicists may think this kind of rigor is not interesting. Do you have some advice for young people who might want to go along a similar route?

We work mostly on foundational issues. Often, we end up proving things that physicists take somewhat for granted simply because it seems so obvious – it somehow seems intuitively to make sense. It's a little bit like building houses. It's always good to know that you are building on a solid foundation, pour enough concrete, distribute the weight of the building. So mathematicians do the groundwork; in many cases there is no shifting sand, but then again there might be.

Take the famous $1 million Clay problem about the Navier–Stokes equations for example. It is a completely mathematical statement and most physicists would say, "Who cares? The answer certainly seems obvious. We don't need the equation to tell us that. We just observe the fluid and it's not going to explode or something." But it isn't known whether the solutions are unique or not. Terence Tao seems to believe they are not for at least some special initial conditions. If it turns out they are not, then these Navier–Stokes equations sometimes simply aren't a good physical description of natural fluids. If we can't prove that the equations don't have this strange behavior, what it would tell you is that they are kind of wrong in the first place.

Did you expect to win the Fields Medal this time?

I knew I was being considered. Everybody was talking about a dozen or so people being considered. But I didn't think I had a very realistic chance of getting it, so it certainly came as a surprise.

You were informed beforehand. How did you keep it a secret?

Actually, people were very nice about not trying to extract information from me. Generally, most mathematicians are quite considerate people.

SEOUL ICM 2014

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Meaning of "Math&Presso" A compound word joining "math" and "press" with a slight alteration and pronounced similar to "espresso," this newsletter is distributed to participants every morning and will, we hope, serve our readers like a hot cup of coffee. Math + Press = Math&Presso ≈ Espresso

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The Organizing Committee of the International Congress of Mathematicians has commissioned the Korea JoongAng Daily, the leading English-language newspaper in Korea, to publish an official daily newspaper for the congress. The editors and the organizers are not responsible for the opinions expressed by the interviewees and published here or for any consequences arising from the accuracy of the information herein.

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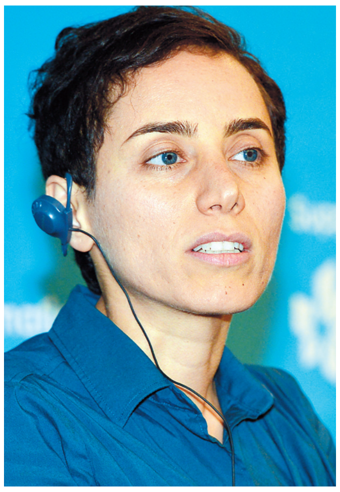
Maryam Mirzakhani: Fields Medalist

Moduli spaces await her forthcoming discoveries



Maryam Mirzakhani speaking at a press conference Wednesday after the SEOUL ICM 2014 opening ceremony, where she was awarded a 2014 Fields Medal.

Maryam Mirzakhani has made striking and highly original contributions to geometry and dynamical systems. Her work on Riemann surfaces and their moduli spaces bridges several mathematical disciplines and influences them all in return. She gained widespread recognition for her early results in hyperbolic geometry, and her most recent work constitutes a major advance in dynamical systems.

A surface, which is a two-dimensional topological manifold, becomes a *Riemann surface* when it is endowed with an additional complex structure which allows one to do complex analysis on the abstract surface. An alternative but equivalent way of defining a Riemann surface is through the introduction of a geometry that allows one to measure angles, lengths, and areas. The most important such geometry is *hyperbolic geometry*, the original example of a non-Euclidean geometry discovered by Bolyai, Gauss, and Lobatchevski. The equivalence between complex algebraic and hyperbolic structures on surfaces is at the root of the rich theory of Riemann surfaces.

Mirzakhani's early work concerns closed geodesics on a hyperbolic surface. A now-classic theorem, the so-called prime number theorem for geodesics, says that the number of closed geodesics whose length is bounded by L, grows exponentially with L; specifically, it is asymptotic to e^L/L for large L. She looked at what happens to this theorem when one considers only the *simple* closed geodesics, meaning that they do not intersect themselves. The behavior is very different in this case: the growth of the number of simple geodesics of length at most L is of the order of L^{6g-6} where g is the genus of the surface. Mirzakhani showed that in fact the number is asymptotic to $c{\cdot}L^{6g-6}$ for large L (going to infinity), where the constant c depends only on the hyperbolic structure.

While this is a statement about a single, though arbitrary, hyperbolic structure on a surface, Mirzakhani has also studied the *moduli space* considering all such structures simultaneously. She first established a link between the certain volume calculations on a moduli space and the counting problem for simple closed geodesics on a single surface. Later, she and her co-workers proved the theorem by counting closed geodesics in moduli space, inspired by the work of Margulis. She has also proved that certain dynamical systems on moduli space, known as the *earthquake flow*, is chaotic.

Most recently, Mirzakhani, together with Alex Eskin and, in part, Amir Mohammadi, made a major breakthrough in understanding another dynamical system on moduli space that is related to the behavior of geodesics in moduli space. Non-closed geodesics in moduli space are very erratic and even pathological, and it is hard to understand how they change when perturbed slightly. However, Mirzakhani et al. have proved that *complex* geodesics and their closures in moduli space are in fact surprisingly regular, rather than irregular or fractal.

Although moduli space has complexities and inhomogeneity, Mirzakhani has a strong geometric intuition that allows her to grapple directly with the geometry of moduli space. Fluent in a remarkably diverse range of mathematical techniques and disparate mathematical cultures, she embodies a rare combination of superb technical ability, bold ambition, far-reaching vision, and deep curiosity.

Moduli space is a world in which many new territories await discovery. Mirzakhani is sure to remain a leader as the explorations continue.

Extract from the IMU news release

Tuesday, August 19

09:00 - 12:30	Plenary Lectures	Hall D1
	09:00 - 10:00 *Analytic Low-Dimensional Dynamics: from dimension one to two*	PL-13
	Mikhail Lyubich, Stony Brook University, USA	
	10:15 - 11:15 *Approximate algebraic structure*	PL-14
	Ben Green, University of Oxford, UK	
	11:30 - 12:30 *Some mathematical aspects of tumor growth and therapy*	PL-15
	Benoît Perthame, Université Pierre et Marie Curie, France	
12:30 - 14:00	Lunch	
14:00 - 15:00	Fields Medalist Lecture by Maryam Mirzakhani has been canceled	Hall D1
15:00 - 18:00	Invited Section Lectures	
	3. Number Theory	307ABC
	5. Geometry *	Hall E1-4
	7. Lie Theory and Generalizations	318ABC
	9. Dynamical Systems and Ordinary Differential Equations	308ABC
	10. Partial Differential Equations *	300
	11. Mathematical Physics	317ABC
	Invited Section Lecture [IL11.7] by Robert Seiringer scheduled at 15:00 has been canceled.	
	13. Combinatorics	Hall E5-6
	14. Mathematical Aspects of Computer Science	402
	17. Mathematics in Science and Technology *	327ABC
	19. History of Mathematics	301AB
	* ends at 18:45	
15:00 - 16:40	Invited ICM Panel Discussion: R&D policy (ERC+NRF)	Hall D1
15:00 - 18:00	Short Communications	
	2. Algebra	309
	3. Number Theory	310AB
	4. Algebraic and Complex Geometry *	311AB
	5. Geometry	316
	6. Topology	312
	8. Analysis and its Applications	320AB
	9. Dynamical Systems and Ordinary Differential Equations	324AB
	10. Partial Differential Equations	319
	12. Probability and Statistics	322
	13. Combinatorics	321AB
	15. Numerical Analysis and Scientific Computing	325AB
	18. Mathematics Education and Popularization of Mathematics	326
	19. History of Mathematics *	301AB
	* ends at 19:00	

17:00 - 20:00	London Mathematical Society Reception	
	17:00 – 18:30 Meeting	101
	18:30 – 20:00 Reception	102
12:00 - 18:00	Poster Session	Hall C1
	16. Control Theory and Optimization	
	17. Mathematics in Science and Technology	
	18. Mathematics Education and Popularization of Mathematics	
18:00 - 19:30	IMU Panel Discussion: Future of Publishing	402
19:00 - 21:00	Japan Reception	104
19:30 - 21:30	Brazil Reception	105

Other Activities

15:00 - 16:05	Public Baduk (Go) Lecture	401
16:10 - 18:00	Public Baduk (Go) Event	401 Lobby
17:30 - 20:30	Math Movie Screening, "How I Came to Hate Maths"	Hall D1

The list of members of all 2014 ICM prize committees can be found in the Program Book, Page 200, and will also be published in Vol. 1 of the Proceedings.

Artur Avila Autograph Session

Fields Medalist Artur Avila will be signing autographs and taking pictures at the ICM 2018 booth in the exhibitors hall (Hall C1) on Tuesday, August 19, from 2pm to 3pm.

Daily Math Puzzle

Q. Find the largest integer that is a product of a three-digit number and a one-digit number and also a product of two two-digit numbers.

□□□ × □ = □□ × □□